Ch1 - Math

多元微积分的应用，无需多言，记得根据自己想要求的对象选择适当**原点**和**座标系类型记得积边缘的时候要注意无穷小的阶级。积范围内的面积/体积时，多出来的误差部分随着$x \to 0$变成高阶无穷小，积分的时候直接忽略。但对于边缘（边缘长度），需要用勾股定理计算。

[Definition - differential elements]

1. differential **length** element dx - a infinitesimal increment in x
2. differential **surface** element $dS = dxdy$ - differential square where we have a infinitesimal increment in both x and y
3. differential **surface area** element $\vec{dS} = dxdy\hat{n}$ - magnitude given by surface area, direction given by the normal vector of the area
4. differential **volume** element dV = dxdydz - A differential cube where we have a infinitesimal increment in x, y, and z

这是在物理中的含义，跟数学上的differential有区别

Ch2 - Electrostatics

electron has negative charge, neuron has positive charge

$$m_p \approx m_n \approx 1.7e{-}27kg$$

$$m_e \approx 9.1e{-}31kg$$

$$Q_e = -Q_n = -1.6e{-}19C$$

毛皮摩擦琥珀，琥珀电子被夺走带正电，毛皮得到电子带负电 - 这种现象叫做**triboelectric effect**

Conservation of Charge

The total charge in an isolated system remains the same. We cannot create or destroy charge, but we can transfer it.

> KCL就是基于Conservation of Charge

Quantization of Charge

Charge must be integral multiple of the fundamental charge of electron. (电荷量是离散的，根据电子的电荷来)

Coulomb's Law

Interaction between charges obeys Coulomb's law.

Coulomb's law defines electrostatic forces.

$$\vec{F}_{Qq} = k\frac{Qq}{r^2}\hat{r}_{Qq} = -k\frac{Qq}{r^2}\hat{r}_{qQ}$$

互相之间的力大小相等方向相反，库伦定律的到的力的正负表示力的方向

$k = 1/4\pi\epsilon_0 \approx 8.99e9 Nm^2/C^2$

ϵ is the **permittivity**. $\epsilon_0 = 8.854e{-12}F/m$ is the permittivity of *free space*(space free of induced charges)

The ϵ we use will change if we change the medium between charges. ϵ_0是常量

电荷的单位Coulomb由电流定义, 1C of charge is the charge transported by 1A of current in 1s.

$1C = 1A * 1s$

Electrostatic force follows the Superposition Principal

Limitations of Coulomb's Law

Coulomb's Law is the Newtonian way of doing physics.

库伦定律是猜想，并没有证明，而现在的实验已经知道了:

1. 库仑定律对$r \in (10^{-16}m, 10^8m)$范围内都生效
2. 并且r的次方$\in 2 \pm 10^{-16}$, 基本锁定就是平方反比

库仑定律只适用于静电力, electrostatics

Ch3 - Electric Fields

Electric field is a *vector field quantity*

> A vector field is a function which assigns a vector value (magnitude and direction) to every point in space

[Definition - Electric Field] Electric field due to Q at point P is the **force per unit** test **charge** that is very small (direction is the dir. of force experienced by **positive** charge)

$\vec{E}(\vec{r}) = lim_{q\to 0}(\vec{F}_Q/q)$

对于点电荷

$\vec{E}(\vec{r}) = \vec{F}_Q/q = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}[N/C]$

> 数学上的单位是N/C，工程上的单位是V/m

电场的叠加遵循Superposition Principle，某一点的电场等于所有电荷电场的总和

电场的图像 - 电场线 - 电场线之间的距离越小，该处电场越强 (电场也可以画箭头表示)

> Convention - The number of lines per unit area at right angles to the lines is proportional to the field strength
>
> 电场线/箭头的疏密与电场强度呈正比

> 看电场图像判断电荷
>
> 如果出现dipole，则是正电荷指向负电荷，直接连接(出现直接连接或者画弧线一圈回到原来的地方都是dipole)

> 如果两种相同电荷，则是近似直角的十字路口

Electric diplole

Two charges separated by a small distance with equal value but opposite polarity

假如两个电荷距离为d，则当$r >> d$时，$\vec{E}_P(\vec{r}) = kq\frac{2d}{r^3}\hat{r}$

> dipole周围的电场强度与距离成立方反比，下降的更快

因此，$E \propto \frac{qd}{r^3}$

因此我们定义dipole moment $\vec{p} = q\vec{d}$

> 这是物理的表达方法，化学的可以上网看

Ch4 - Calculation of Electric Fields

解题思路：

1. 找charge density(ρ)，若没给且电荷是uniform，则自己算出来 (具体问题具体分析，若不是uniform distributed charge, 必要时要直接通过题的表达式积)
2. 找到合适的座标系和原点
3. 找一个点去分析那个点的电场表达式
4. [重要]一定记得电场是矢量，所以不可以直接积分，找对称，看看能不能变成标量，把大小先积出来。如果没有对称，则在座标系的每个座标上分别积分

Ch5 - Gauss Law

Electric Flux

大概概念就是说想象一个电灯泡，发出的光是恒定的，不过我们如何去quantify光的量？我们用小球把灯泡包裹起来，和大球包裹起来，照在球面上的光应该是一样多的。Flux想做到的就是把这个概念用数学表达出来

Differential flux of field

$d\phi_e = \vec{E} \cdot \vec{dS}$

> 记得differential surface element $\vec{dS} = dS \cdot \hat{n}$

积分得到**flux of electric field**

$\phi_e = \int_{surface} \vec{E} \cdot \vec{dS}$

Displacement Vector

在物理学中为了方便计算

$\vec{D} = \epsilon_0 \vec{E}$

真正的**Electric Flux -** Integral of the dot product of displacement vector and differential surface area element

$\epsilon_e = \int_{surface} \vec{D} \cdot \vec{dS} = \int_{surface} \epsilon_0 \vec{E} \cdot \vec{dS}$

Electric Flux over closed surface

想象一个点电荷在球面中

可以通过积分得到

$\phi_e = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \frac{\epsilon_0 Q}{4\pi\epsilon_0} sin\theta d\theta d\varphi = Q$

Charge outside a closed surface

根据convention, closed surface $\vec{dS}$ always points normal to the surface **away** from the enclosed volume

所以在外面电荷产生的电场一进一出相当于没有，$\phi_e = 0$

The electric flux due to a charge outside the closed surface will always be 0, no matter how the charge is placed, no matter what the shape of the charge is, and what the shape of the surface is.

Gauss Law

假如已经给了一个closed surface, 那么我在其外面套一层薄薄的surface，然后挖空原来surface，那么这层薄薄的volume的flux为0(假设这里面没有电荷)，因为这个flux里面没有电荷。外面套的surface未必要是sphere，可以是任意形状，因此基于Charge outside a closed surface will always be 0，我们有了高斯定律：

Total flux through a closed surface is only equal to the total charge enclosed within the volume of the closed surface, no matter how the charge is distributed, no matter what the shape of the charge is and no matter what the shape of the surface is provided as long as it is closed.

$\oint \vec{D} \cdot \vec{dS} = \oint \epsilon_0 \vec{E} \cdot \vec{dS} = Q_{enclosed}$

> 高斯定律唯一条件就是closed surface
>
> Gauss law is nothing but a mathematical restatement of the square law of Electric fields we have seen before, but provides a new perspective of looking at fields.
>
> 高斯定律原理就是平方反比，就像球壳内的电场处处为0一样

> 来自习题: 电荷不能通过电场来固定位置
> 假如你要把一个正电荷固定在一个位置，那么那个位置必须要有指向那个位置的电场。这时你做一个closed surface，由gauss law，那个surface里面必须要有负电荷，而你想固定正电荷矛盾了，所以电荷不能用电场固定

Calculating Electric Field with Gauss Law

Gauss law on its own can never calculate electric field

We need to find a special symmetry (call it Gaussian symmetry, but it does not have an official name)

在这个Surface上，E_n 处处相等且normal to the surface，或者electric flux等于0(field tangential to the surface)

> 每个穿过平面的E-field要么normal，要么tangential
>
> normal的E_n处处相等，tangential的E_n没有限制
>
> 为了满足处处相等的条件，判断时可以想象在surface上的各个地方看charge distribution, charge distribution必须处处都看起来一样
>
> 这也就限制了用gauss law计算电场只适用圆的电荷分布，或者无限长的电荷分布
>
> [注意]Gaussian symmetry 不代表电荷必须几何对称分部，只要能保证从各个点受到的电场强度相同且垂直于平面即可

这样就可以将E_n作为常数从积分中拿出来，如果由垂直于E-field的部分，就可以将那部分面积直接忽略，不积分

$E_n \oint dS = Q_{enclosed}/\epsilon_0$

Sphere

使用球形surface

Charge density ρ can only be a function of r, not φ nor θ - $\rho = \rho(r)$

这样才能做到从球面surface上看charge的分布都一样

球内部的磁场只跟内部的小球的电荷有关，球内部surface之外的部分都互相抵消掉了，参考地球磁场，可以用积分证出来

Infinite Line

使用圆柱形surface

圆柱的两个盖与我们知道的电场方向垂直，且从圆柱表面看两边都是无穷远，normal方向互相抵消

导线上的电荷必须均匀分布

$\vec{E} = \frac{\rho_0}{2\pi\epsilon_0 r}\hat{r}$

Infinite Sheet

使用直棱柱surface(圆柱，直四棱柱, ...)

sheet上的电荷必须均匀分布

棱柱侧面全都与我们想求的电场垂直，normal方向抵消

顶面与底面上看电场分布完全相同

$E = \rho_0/2\epsilon_0$

注意与r无关，infinite sheet 的电场是匀强电场

因此，两个电性相反电极板的之间电场会是匀强电场，并且极板外的电场完美抵消

Infinite Slab(Thick infinite sheet)

Slab 上电荷每层sheet的分布必须相同(把slab想象成无穷多个sheet)

也使用直棱柱做surface

棱柱的侧棱与电场平行，不用管

而棱柱的底面和顶面会有相同的电场 - 每层sheet给两个面带来的电场强度是相同的，因为是infinite sheet

[注意]这种情况下，在顶面和底面看电荷分布会不同，但是我们用了infinite sheet的结论，所以最后的E-field依旧是处处相同的，可以从积分中当作常量拆出来

Special Case

1. Use superposition when the charge can be broken in to multiple Gaussian symmetry parts
2. When we know a region that E-field = 0

> Case 2可以用于p-n junctions, MOS capacitors, or where anti-symmetry exists(比如相反电性，相同电荷量的电极板)
>
> 对于相反电性相同电荷量的电极板，由于极板外面电场抵消，棱柱surface只有底面由normal的E-field, 所以连superposition都永不上，直接就能求得$E = \rho_0/\epsilon_0$

Ch6 - Electrostatic Potential

Mathematical Definition

Electric potential energy - 把电荷从无穷远出运过来要做的功

$W_I = -\int_{point\infty}^{P} q\vec{E} \cdot \vec{dl}$

Electric potential - work I do per unit of **positive** test charge that is very small

$V_P = lim_{q\to 0}(W_I/q) = -\int_{point\infty}^{P} \vec{E} \cdot \vec{dl}[V]$

> Electric potential的定义是我做的功，所以对于负电荷，我会做负功，所以是负的potential
>
> 通常reference是无穷远处，不过通常我们只关心两个点的电势差

Electrostatic fields are **conservative fields** - it does not matter what path we take to reach point P, we will always have the same potential

注意一个比较特别的结论 - 从一个点无论怎么走，最终回到原点的电势差都是0

$\Delta V_{A\to A} = -\oint \vec{E}\cdot\vec{dl} = 0$

这就是KVL，不过注意，这个只适用于electrostatic field, 所以KVL不适用与AC电路，Faraday's Law 开始发挥作用

一般计算电势时，我们会尽量减少数学上的复杂度，所以我们一般沿着electric field line进行积分

点电荷的电势

首先，$dV = -\vec{E}\cdot\vec{dl}$

以电荷为原点建立polar coordinates

这样$\vec{dl} = \vec{dr'}$, 我们选择电场线来积分，电场线是直线，所以矢量可以直接变成scalar

$V_P = -\int_\infty^r Edr = -\int_\infty^r \frac{Q}{4\pi\epsilon_0 r'^2} = \frac{Q}{4\pi\epsilon_0 r}$

复杂电荷分布的电势

计算多个电荷的电势时由两种思路

1. 计算沿一条磁感线上的所有电场，然后用定义积分(通常用高斯定律处理)
2. 使用Superposition Principle，对那个点计算所有电荷的电势，再叠加，因为电势是scalar，所以不用考虑矢量方向问题
   $V_P = \int \frac{\rho_v dv}{4\pi\epsilon_0 r}$

注意两种思路本质上的区别，一种是积电场，第二种是积(已经积分之后的)电势

一般做题流程就是先试试能不能用Gauss Law, 如果不能再用第二种

Electrostatic Potential Energy

$\Delta U_e = qV_P$

> Electric potential can be considered as the ability in the system to gain/loose energy if a new charge is brought into the system

Potential Energy in a System of Charges

每次增加一个charge，总势能都会增加$q_{new}V_{new_p}$

而在新增加的点new_p处的势能又等于所有已经在系统中的电荷的电势相加，所以

系统的总电势能等于系统中所有pair of 电荷的电势相加

$U = k\Sigma_{i=1}^N \Sigma_{j>1}^N \frac{Q_iQ_j}{r_{ij}}$

Electric Potential Between Points

$V_{A\to B} = -\int_A^B \vec{E}\cdot\vec{dl} = -(\int_\infty^B \vec{E}\cdot\vec{dl} - \int_\infty^A \vec{E}\cdot\vec{dl}) = V_B - V_A$

Electric Field from Electric Potential

$E = -\nabla V$

> 关于Nabla
>
> When del operator ∇ operates on a scalar and creates a vector -> It's called a **gradient operation**
>
> $\nabla = [\frac{\partial}{\partial x}\hat{x}, \frac{\partial}{\partial y}\hat{y}, \frac{\partial}{\partial z}\hat{z}]^T$
>
> When del operator ∇operates on a vector, we could either use dot product or cross product
>
> When we use dot product, this is called the **divergence operation**
>
> $\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
>
> ($\vec{A}$是向量，这里的A_x, A_y, A_z不是偏微分，而是矢量在那个方向的Component)
>
> When we use cross product, it is called the **curl operation**，计算漩涡时很有用
>
> $\nabla\times\vec{A}$

Equipotential Surfaces

关于Electric Potential的level curve

正如level curve总跟gradient vector垂直, equipotential surface总跟e-field line垂直

或者可以参考$V_{A\to B} = -\int_A^B \vec{E}\cdot\vec{dl}$, 因为是点乘积，所以如果在equipotential surface走的话，电场只能垂直于我行走的方向

Ch7 - Conductors

Ideal Conductors

Materials that have the following properties

1. There are **free** electrons that are very weakly bound to the atoms.
2. There's no resistance to the motion of those **free** electrons. (no resistance)
3. There are **infinite free electrons** available
4. Electrons will only move **within** the conductor, they will not leave the conductor

Conductors inside external Electric fields

1. Electric Field inside a conductor is always 0

2. Conductors are equipotential (跟上面说的一回事)

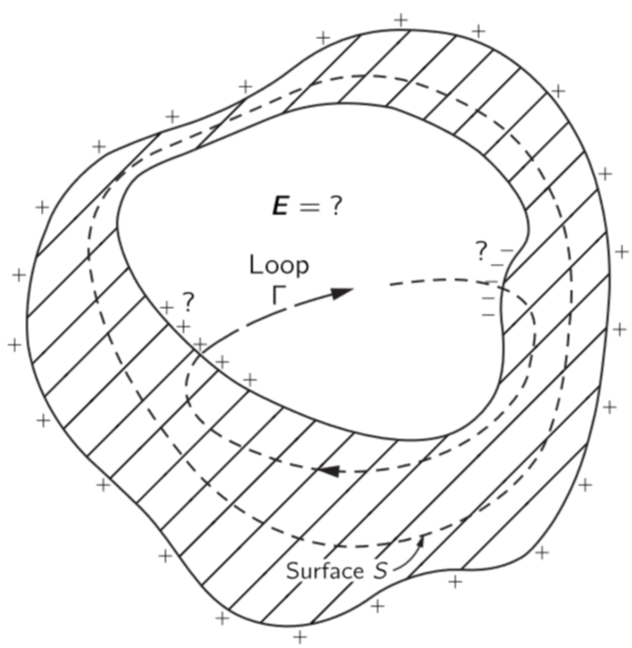
> 电子会重新排布，让导体内部的电场为0，这个过程一般在几个femto(1e-15) seconds中实现
> 这也是让理想导体内部有无穷多自由电荷的原因，否则太强的电场没有足够的电荷去抵消

3. Charges will be induced on the **surface** of the conductor
4. There is no induced charge in the **volume** of the conductor

> 这两个可以通过高斯定律证明，或者换一种思路，如果有电荷，那么就会影响其电场分布，导致这个电荷被排斥/吸附到导体表面

Faraday's Cage

Electric field due to External Electric field inside a cavity will be 0.



由于conductor内部没有电场，所以Surface S内部静电荷为0

现在假设在内表面的电荷分布不同，左边+，右边-，那么一定存在一条电场线，从正到负，这样会导致导体的内表面不等势。

因此内表面必须呈电中性。

中空导体过滤电场

假如上面的图中空心部分任意位置有一个电荷，那么Surface S内部静电荷为0，所以内表面的电荷量与空心部分相同。

剩下的电荷跑到外表面，而因为内表面跟里面的电荷抵消了，所以外表面上的电荷排布完全不受里面电荷位置的影响。

外表面的电荷量受内部电荷量影响，电荷排布受到外表面形状影响，仅此而已。

> 电场经球形中空导体过滤后，对外的电场类似于点电荷的电场

导体接地与内部电场屏蔽

接地会将导体的**电势**拉到0，不是电荷！！！

当中空导体接地后，为了保持电中性，外表面要呈电中性。这样对外来说，从无穷远处积分过来的电势为0（因为没有电场）。

处理多层导体分别接地的问题时，可以考虑哪个导体电势要为0，考虑电势关于距离的图像，看看怎么样分布电荷才能让这个电势降下来。

> 警惕无穷大的平面
>
> 当出现两片infinite sheets时，整个空间会被划分成两个部分，两边的无穷远被隔开了，所以可以说，两边时不同的无穷远，而此时分析时只能选择一边的无穷远来分析所有位置

电荷在导体表面的分布

Note里面给的例子我觉得不太好，基本结论是高斯曲率越高的地方，电荷密度越大。电荷密度与高斯曲率时不成直接函数的对应关系。$\sigma = \sigma(\kappa)$

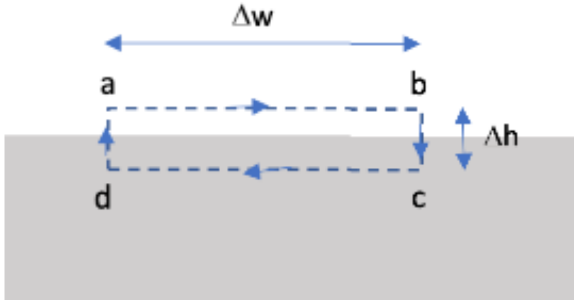
> 当形状凹进去时，高斯曲率为负，所以电荷密度变小

可以从能量的角度入手，电荷之间有相互作用力，所以电荷的排布要尽可能的降低势能，所以每个电荷尽量离的越远越好

导体表面的电场 - Surface Conditions

Any External Electric field lines will have to be perpendicular to the conductor surface; there cannot be a tangential component of the field lines.

Conductors in external electric fields will bend fields so that they hit the conductor (incident on the conductor) perpendicularly(normally).



可以看到，如果导体表面存在切向电场的话，那么这个loop上的电势就不为0了，因此进入导体的电场线必定是垂直于它的（导体等势面，电场线必然与等势面垂直）

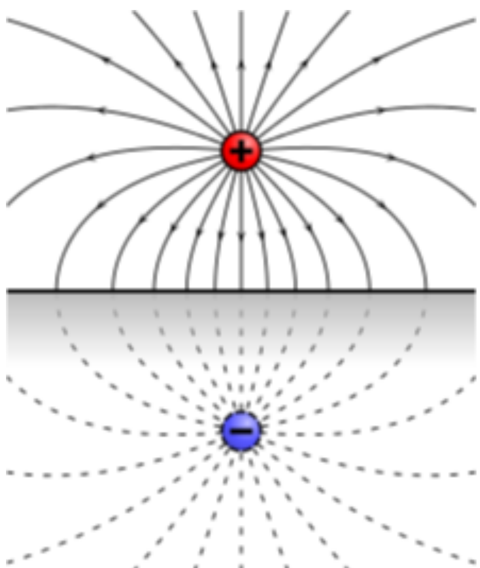
那么我们可以贴着导体表面做高斯曲面，这样导体周围电场强度也能算出来

$E_n = -\frac{\rho_s}{\epsilon_0}$

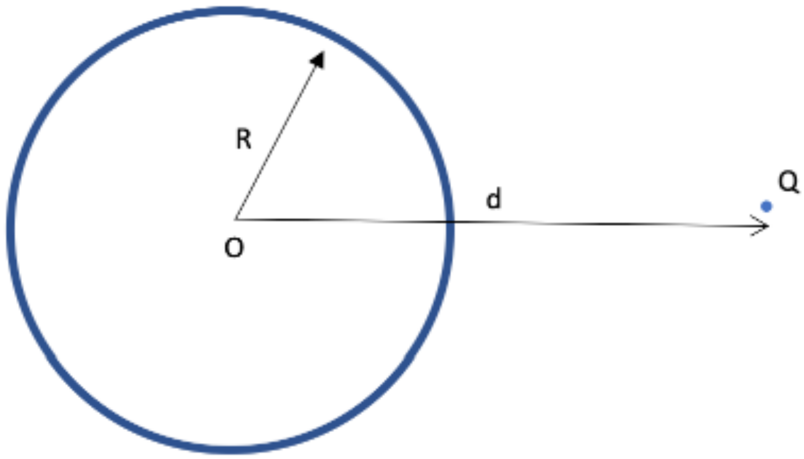
*Uniqueness Theorem and Method of Images

The **uniqueness theorem** guarantees that there is **only one** function, V(r), which describes the potential in that region. Since electric field is gradient of electric potential, this also guarantees there is **only one** function, E(r), which describes the electric field in the region.

> 如果我们找到了合适的电场线排布，那它肯定是我们想要的，而且唯一



求电势的小技巧



这样一个点电荷和一个球形导体的排布，求球形导体的电势时可以只求其圆心的电势，因为球形中空导体内部全是等势的

为什么不能选任意一点而是选圆心?

其它点也是等势的，但不方便计算，选圆心后，圆心的电势可以用积分求

$V_{(r=0)} = \int_{sphere} \frac{kdQ}{R} + \frac{kQ}{d} = \frac{k}{R} \int dQ + \frac{kQ}{d} = \frac{kQ}{d}$

整个圆环上的电荷产生的电势，以及点电荷产生的电势。

而因为圆环上对圆心R处处相等，所以可以提出积分简化计算

Ch8 - Dielectric Materials

When external electric field is applied to an insulator, the electrons in the atom are displaced towards one side.

The insulator is said to be "polarized". And **when we consider the polarizability of the insulator**, we refer to them as **Dielectrics**

Polarization Vector

对于一个dipole，我们用dipole moment来表示它的特性$\vec{p} = q\vec{d}$

Polarization Vector则是某一点上volume density of the electric dipole moment

$\vec{P} = lim_{\Delta v \to 0} \frac{\Sigma_{i=0}^{n\Delta v} \vec{p}_i}{\Delta v}$

Induced Surface charge $\rho_s = \vec{P} \cdot \hat{n}$

Induced Volume charge $\rho_v = -div\ \vec{P} = -\nabla \vec{P}$

> divergence表示的是net outward flux of a point

Derivation of displacement vector

之前的Gauss Law

$\oint_S \vec{E} \cdot \vec{dS} = \frac{Q_{enc}}{\epsilon_0}$

转换成divergence的模式

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_{free}+\rho_v}{\epsilon_0} = \frac{\rho_{free}-\nabla\cdot\vec{P}}{\epsilon_0}$

$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_{free}$

因此，我们定义$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

这样，我们就能得到游离电荷了

$\nabla \cdot \vec{D} = \rho_{free}$

$\oint_S \vec{D} \cdot \vec{dS} = Q_{enc}^{free}$

Permittivity

When the dielectric properites of the medium are **linear** and **isotropic**(doesn't change with direction), the **polarization vector** will be directly **proportional** to the **electrical field intensity**.

$\vec{P} = \epsilon_0 \chi_e \vec{E}$

χ_e is a **dimensionless** quantity called **electric susceptibility**

A dielectric medium is **linear** if χ_e is **independent of $\vec{E}$**, and is **homogeneous** if chi_e is **independent of space co-ordinates**

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} = \epsilon_0 (1+\chi_e) \vec{E} = \epsilon_0 \epsilon_r \vec{E} = \epsilon \vec{E}$$

relative permittivity(dielectric constant): $\epsilon_r = 1 + \chi_e$, measures how strongly does the material polarize with respect to free space

absolute permittivity: $\epsilon = \epsilon_0 \epsilon_r$ Unit: Farads/m,

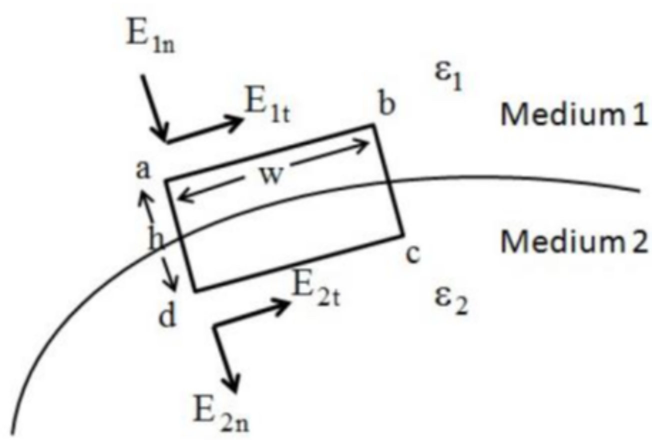
那么反过来也可以计算Polarization Vector

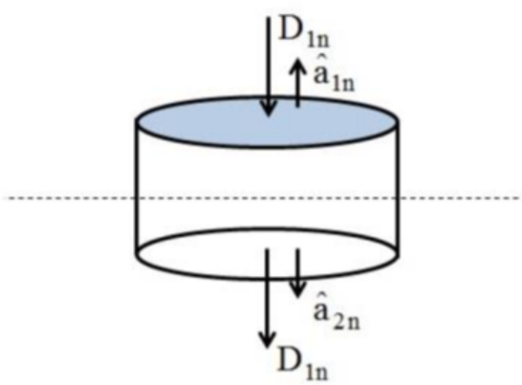
$\vec{P} = \vec{D} - \epsilon_0 \vec{E} = \epsilon_0 \epsilon_r \vec{E} - \epsilon_0 \vec{E} = \epsilon_0 (\epsilon_r - 1) \vec{E}$

> 由于电场作为field的性质，空间内每个点的场强只跟那个点的电介质有关，跟电介质怎么排布的，隔了几层电介质无关。
>
> 理想导体的relative permittivity?
>
> 由于在理想导体中，电子polarize的程度极其剧烈，因此$\epsilon_r \to \infty$

> 注：实际上导体的relative permitivity是个复数，但不在这门课的范围之内了

Boundry Condition





> 平行于电介质表面电场相同
>
> 垂直于电介质表面，displacement vector相同

Tangential Boundry Condition: $E_{1t} = E_{2t}$

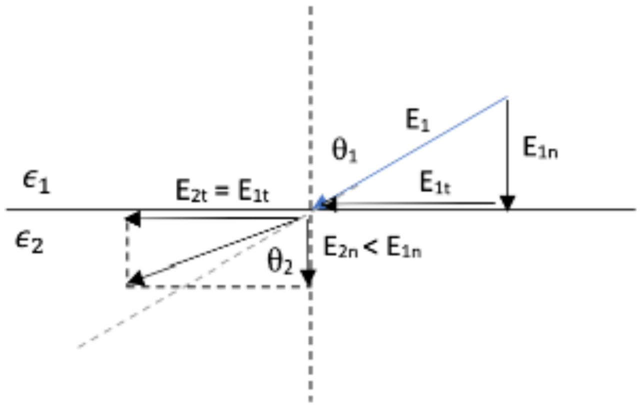
The tangential components of the electric fields have to be equal across the boundry

Normal Boundary Condition: $\epsilon_2 E_{2n} = \epsilon_1 E_{1n}$

$D_{2n}A - D_{1n}A = \rho_s^{free} A = 0$

$D_{2n} = D_{1n}$

> 介电常数大的，导电性越好，电场越小
>
> 介电常数小的，绝缘性越好，电场越大



$E_{1t} = E_{2t} = E_1 sin(\theta_1)$

$E_{1n} = E_1 cos(\theta_1) = \frac{\epsilon_2}{\epsilon_1} E_{2n}$

根据上面这两个关系也可以得出入射角与反射角关系

$\epsilon_1 tan(\theta_2) = \epsilon_2 tan(\theta_1)$

Ch9 - Capacitance

The capacitance is the electric charge that must be added to the conducting body to increase its electric potential by 1V.

Q = CV

C is defined as Coulomb/Volt, or Farad(F)

Capacitor

The capacitance of a capacitor is a **physical property** of this system. It **only** depends on the **geometry** of the conductors, **distances** between them, and the **permittivity** of teh dielectric medius between them.

There exist negative capacitance, however, in this course we only deal with positive capacitance.

Calculation of Capacitance

Electric field -> Electric potential -> Capacitance

> 计算平行板电容器时用到的近似 - Field is all confined within the capacitor and the charge is uniformly distributed.
>
> This is not consistent with **boundry conditions** and **conservative nature of electrostatic fields**
>
> 在最外层的电场线如果突然消失，就会导致沿着外围一圈上KVL不成立
>
> 最外圈上会有电场溢出来，不过圆形电极板的溢出是最小的，在最外圈加一层金属也能减小溢出
>
> 当这个电容的scale变小时，溢出电场的影响就会变大，直到溢出的部分无法被忽略

普通电容的计算

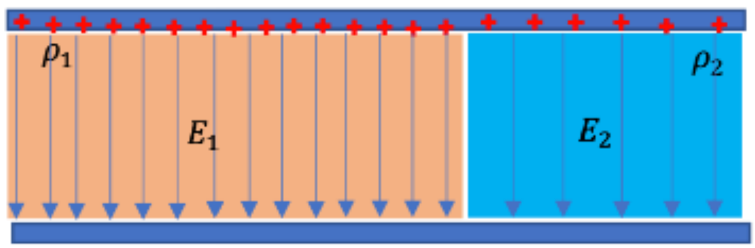
根据高斯定律

$D = \epsilon E = \rho$

$V = Ed = \frac{\rho}{\epsilon} d$

$C = \frac{Q}{V} = \frac{Q\epsilon}{\rho d} = \frac{\epsilon A}{d}$

并联电容的计算



1. 看电场关系

电介质交界处，切向电场相同 - $E_1 = E_2$

因此上方电荷会重新分布

根据高斯定律，$D_1 = \rho_1, D_2 = \rho_2$

$E_1 = \rho_1/\epsilon_1 = \rho_2/\epsilon_2 = E_2$

2. 看电荷关系

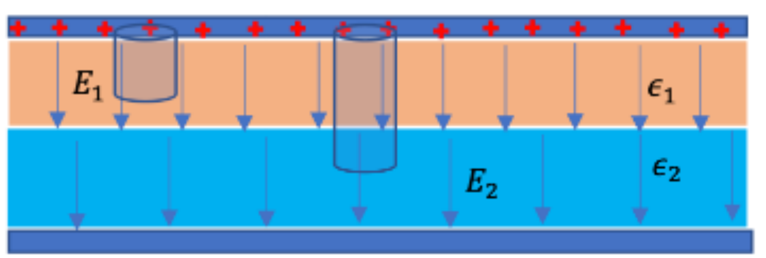
$\rho_1 A_1 + \rho_2 A_2 = Q$ (电荷守恒)

因此能求出ρ_1, ρ_2的表达式，进而求出E的表达式

然后用C = Q/V = Q/Ed 得到

$C = \frac{\epsilon_1 A_1}{d} + \frac{\epsilon_2 A_2}{d}$

串联电容的计算



交界处，垂直方向的displacement vector相同$D_1 = D_2$

$E_1 = \frac{\rho}{\epsilon_1}, E_2 = \frac{\rho}{\epsilon_2}$

$V = \rho d_1/\epsilon_1 + \rho d_2/\epsilon_2$

$C = \frac{\rho A}{\rho(d_1/\epsilon_1 + d_2/\epsilon_2)} = \frac{1}{\frac{d_1}{A\epsilon_1} + \frac{d_2}{A\epsilon_2}}$

Energy Stored in Capacitor

Capacitors are used to store electrical energy in terms of electric fields.

积累能量时，capacitor上含有的电荷越多，新增一个电荷的能量成本就越高，所以要用积分解决

$U_e = W = \int_{q=0}^{Q} \Delta V dq = \int_{q=0}^{Q} \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2}CV^2$

> 移动电容电极板，撤掉电源之类的题目
>
> 看好什么不变，什么变（撤掉电源，电荷不变；不撤电源，电压不变）
>
> 用合适的能量公式找对应关系，熟练运用Q = CV

Energy Density of Electric Field

能量密度 = 单位体积储能

$u_e = \frac{U_e}{V}$

对于平行板电容器的推导：

$C = \frac{\epsilon A}{d}$, $V = Ed$

$u_e = \frac{1}{2}\frac{\epsilon A}{d}(Ed)^2/Ad = \frac{1}{2}\epsilon E^2$

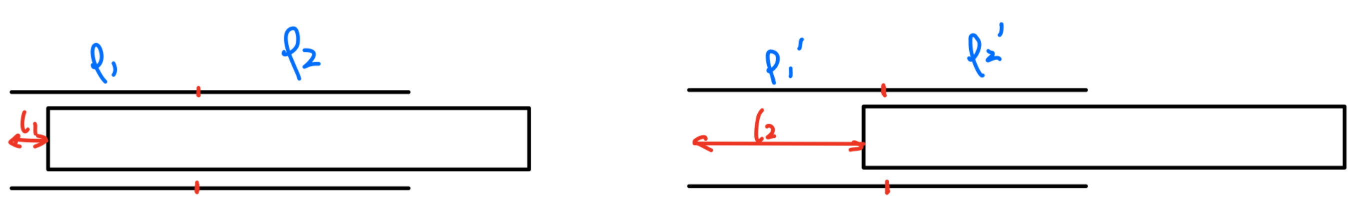
这个式子对于其它电容器也成立

$u_e = \frac{1}{2}\vec{D} \cdot \vec{E} = \frac{1}{2}\epsilon E^2$

推拉电介质问题

在插入和抽取电介质时，电极板上电流方向

在电极板上取一个截面，观察那个界面两边电荷量的变化



如图，在抽取的过程中，$\rho_2' > \rho_2$

因此在抽取的过程中，电荷向右侧移动，电流方向跟抽取电极板的方向相同

Ch10 - Magnetic Fields and Biot Savart Law

New term: Magnetic flux density (Magnetic field) $\vec{B}$, unit: Webers per square meter (Wb/m^2)

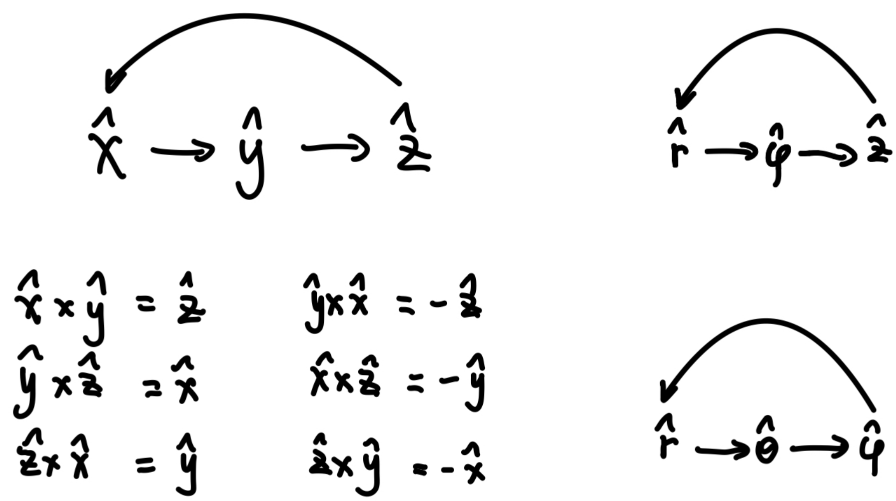
If we have charge q moving at velocity $\vec{u}$ in both Electric field and Magnet field

Lorentz's force equation: $\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$

$\vec{F}_e = q\vec{E}$, $\vec{F}_m = q\vec{u} \times \vec{B}$

Magnetostatics: The field is created by charge moving at uniform velocity, which does not change with time -> the magnetic field does not change with time.

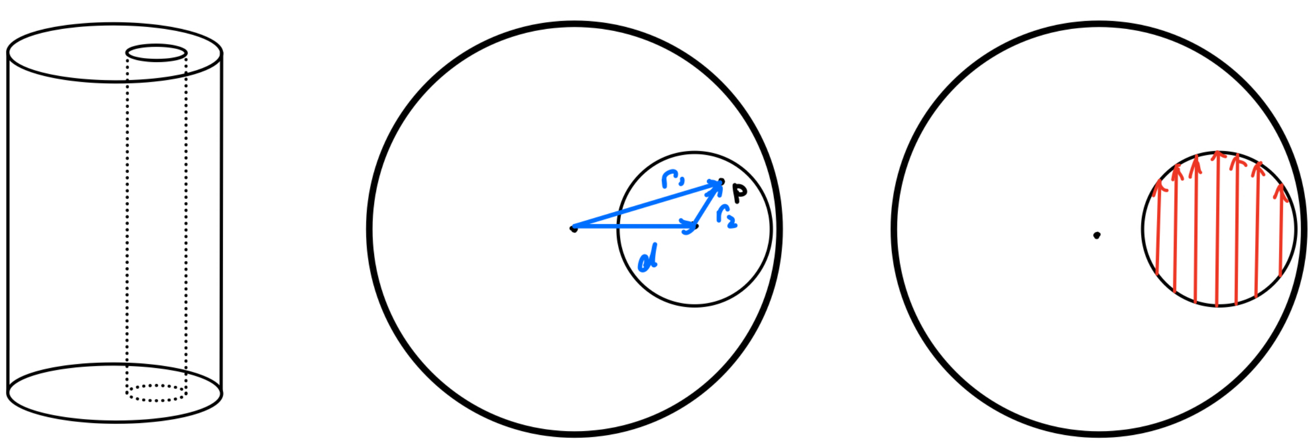
Cross Product[TODO]



[例]极坐标转Cartisian

> 实际上是安倍定律的题，不过放到这里因为跟叉乘相关

中空部分不同轴的圆柱形导线内的电场



利用Superposition计算, 大导线是B_1, 空的部分是B_2

$B_1 = \frac{\mu_0 J_0 \pi r_1^2}{2\pi r_1}\hat{\phi}_1 = \frac{\mu_0 J_0 r_1}{2}\hat{\phi}_1$, $B_2 = \frac{\mu_0 J_0 r_2}{2}\hat{\phi}_2$

这两个分别来自不同的坐标系，有着不同的$\hat{\phi}, \hat{r}$，但他们两个的$\hat{z}$ 相同，因此可以通过数学手段转换

$r_1\hat{\phi}_1 = r_1(-\hat{r} \times \hat{z}) = -\vec{r_1} \times \hat{z}$

$r_2\hat{\phi}_2 = -\vec{r_2} \times \hat{z}$

$\vec{B}_P = \vec{B}_1 - \vec{B}_2 = \frac{\mu_0 J_0}{2}(\vec{r_2} - \vec{r_1}) \times \hat{z} = \frac{\mu_0 J_0}{2}\vec{d} \times \hat{z}$

$B_P = \frac{\mu_0 J_0 d}{2}$

Gauss Law for Magnetism

According to Dirac, there should exist magnetic monopole, but not that we have seen yet.

$\oint_S \vec{B} \cdot \vec{dS} = 0$

Biot Savart Law

$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \hat{r}}{r^2}$

where μ_0 is the **permeability** of free space, $\mu_0 = 4\pi \times 10^{-7} Tm/A$, (unit: tesla meter per ampere)

To get the total magnetic field, we can integrate on the whole circuit.

$\vec{B} = \int_{circuit} \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \hat{r}}{r^2}$

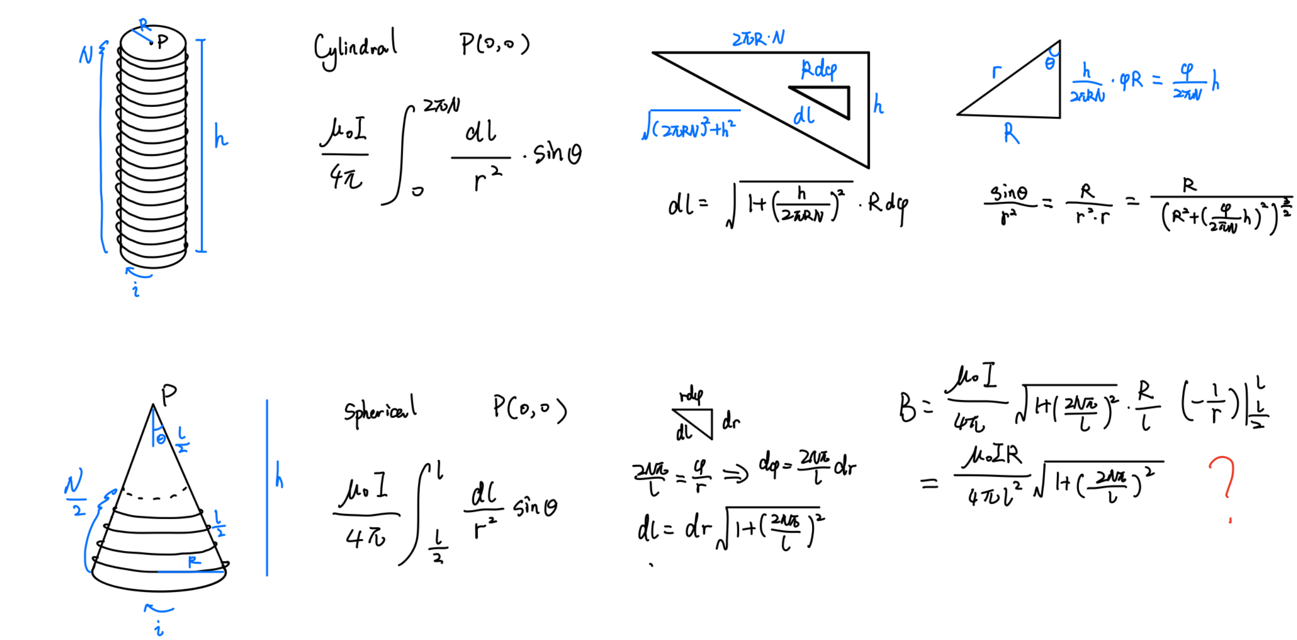
> 都是从线路的角度去分析叉乘的，而不是被测点

For moving charges

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(dq/dt)\vec{dl} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{dq(\vec{dl}/dt) \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{\vec{v} \times \hat{r}}{r^2}dq$$

螺线管的磁场强度(不使用Ampere's Law)

> 如果考虑线的角度，能不能积出来?
>
> 螺线管的积分和离谱，最后用积分计算器得出来的结果是
>
> $B = \frac{\mu_0 NI}{2\sqrt{R^2+l^2}} \cdot \sqrt{1 + (\frac{h}{2\pi NR})^2}$
>
> 后面根号的值随着匝数增加无限接近1



> 下边锥形的计算未必正确，重点在于$d\phi$ 和 dr 的关系，给出一个思路，用作以后参考

需要将匝数N变成单位长度上的匝数$n = N/l$，并且忽略掉线的角度，当成整个面上都有电流

这样对于微分高度dh, 流过的电流就是$nIdh$

$$B = \frac{\mu_0}{4\pi}\int_{solonoid}\frac{nIdh\cdot(2\pi R)\cdot sin\theta}{r^2} = \frac{\mu_0}{4\pi}\int_{solonoid}\frac{nIdh\cdot(2\pi R)}{R^2+h^2}\frac{R}{\sqrt{R^2+h^2}}$$

$$B = \frac{\mu_0 nI}{2}\int_0^l \frac{R^2}{(R^2+h^2)^{3/2}}dh$$

使用换元积分, $h = R\cdot cot(\theta)$

$$B = \frac{\mu_0 nI}{2}\int_{\theta_1}^{\theta_2} -sin(\theta)d\theta = \frac{\mu_0 nI}{2}(cos(\theta_2) - cos(\theta_1))$$

或者

$$B = \frac{\mu_0 nI}{2}\frac{l}{\sqrt{R^2+l^2}} = \frac{\mu_0 NI}{2\sqrt{R^2+l^2}}$$

Ch11 Ampere's Law

> This Ampere's Law is imcomplete, and work only for DC currents and approximately corrent at low-frequencies (quasi-static conditions)

Current Density

为了更好的描述导线上的电流，我们引入新的概念Current Density - J

解决一个问题：无论对导线如何做截面，这个截面上的Current Density Flux都是相同的 $I = \int_S \vec{J}\cdot d\vec{S}$

We define J as a vector where the direction for the vector is along the motion of **positive** charge

$$\vec{J} = nq\vec{u}$$

单位 A/m^2

Ohm's Law (Point form)

$\vec{J} = \sigma\vec{E}$

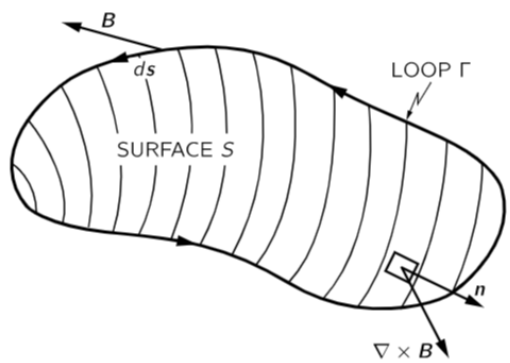
σ叫做conductivity, unit: Seimens per meter(S/m)

从微观角度想，电子在移动时撞到原子核，每过一段时间就会撞到一次，就导致电子有一个最大速度(只是辅助思考，事实上应该比这个复杂)。如果电场大，加速度就大，这样最高速度也会相应变大

conductivity的倒数是resistivity(Ohm/m)

Ampere's Law

穿过非闭合曲面的current density flux与曲面边缘上的电场成正比



$$\oint_C \vec{B} \cdot \vec{dl} = \mu_0 I_{enc} = \mu_0 \int_S \vec{J} \cdot \vec{dS}$$

正因为之前current density的定义，这个非闭合曲面可以是任意曲面，只要loop C不变就行

> 安倍定律的唯一条件是closed loop，没有其它条件
>
> 但是用安倍定律求磁场有条件

如何思考安倍定律：

想象无限长的导线，周围的磁场以1/r的速率降低，然而一个圆的周长是以r的速率增加的。所以类比高斯定律，这个圆无论是什么形状，磁场积分都是相同的。

Calculate Magnetic Field with Ampere's Law

跟Gauss Law求电场大同小异

要求在closed loop上所有位置B相等，或$\vec{B} \cdot \vec{dl}$点乘积等于0（B = 0或与dl垂直）

> 判断方法：在closed loop不同位置观察被测物，必须在所有位置看起来都一样

这样才有可能把B提出来计算，B的方向由右手定则决定(Ampere's Law把方向的信息在积分的过程中消除了)

能用安倍定律计算的形状

- Infinite Cylinders
- Wires having **isotropic** current densities(density doesn't vary with ϕ)
- Infinite sheets with **uniform** current density
- Infinitely long/very long solenoids
- Toroids

> 有限长的导线为何不能用安倍定律算?
>
> 恒稳电流必然是一个环路，所以理论上这种导线不存在
>
> 并且如果这样的话，我更改安倍曲面的位置，导线可能完全没有穿过曲面

Ch12 Faraday's Law

Magnetic Flux

之前都提到磁场Magnetic field又叫Magnetic flux density了，所以无须多言

$\phi_m = \int_S \vec{B} \cdot d\vec{S}$

Unit: 1Weber = 1 Tesla*m

> 记得闭合曲面上的Magnetic Flux恒为0，麦克斯韦方程组的第二条

Faraday's Law

Magnetic Flux变化率等于环路上的电动势

Time varying magnetic flux creates electric field which curls around the magnetic field.

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d\phi_m}{dt} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{S}$$

varying magnetic field energy -> electric energy

Lenz's Law

上面公式中的负号代表的就是楞次定律

感应电流的产生方向与右手定则的方向相反

Electric field is created in such a way that it opposes the rate of change of magnetic flux

> 如果磁场变大，那ROC方向与磁场方向相同，感应出来的电流产生的磁场方向与原磁场方向相反，试图保持原来的磁场强度
>
> 若磁场减小，那ROC方向与磁场方向相反，感应电流的磁场方向与原磁场相同，试图保持原来的磁场强度

可以从能量的角度考虑，如果相同，那么感应电流产生的电场会产生磁场，而这个产生的磁场会加强原来的磁场，左脚踩右脚上天

Electromagnetic Induction

Faraday's Law is non-conservative

环路上会产生电动势，之前的KVL被推翻$\oint_C \vec{E} \cdot \vec{dl} = 0$

$V_{ind} = \oint_C \vec{E} \cdot \vec{dl} = -\frac{d\phi_m}{dt} = -\frac{d}{dt} \int_S \vec{B} \cdot \vec{dS}$

Types of Induced EMF

Transformer EMF

感应线圈不动，由其它导线产生的磁场随导线的电流变化，产生EMF

Motional EMF

感应线圈通过转动，平移，形变等方式改变穿过的magnetic flux

> 通过洛伦兹力理解Motional EMF:
>
> 电荷在磁场中的受力:$\vec{F}_m = q(\vec{u} \times \vec{B})$
>
> 而之前我们定义了电场$\vec{E} = \frac{\vec{F}_m}{q} = \vec{u} \times \vec{B}$
>
> 上面就是感应电场的表达式，可以代入$V_{ind} = \oint_C \vec{E} \cdot \vec{dl}$求电压
>
> 想象一个金属棒在垂直于磁场方向移动
>
> 此时内部的电子就会受力，聚集到一端（质子动不了）
>
> 此时金属棒两端产生电压，而当导体两端电荷的吸引力与磁场产生的力相同时，这个系统达到平衡，平衡状态的电场就是上面的式子，此时金属棒两端电压就是当前的EMF

Flux Linkage

因为我们的线圈通常不会只是一圈，而是很多圈，所以我们要Flux Linkage帮我们处理多匝数线圈的问题

There is no specific definition of Flux Linkage

$V_{ind} = -\frac{d\Lambda}{dt}$

$\Lambda = N\phi_m$

N是线圈的匝数

注意线圈的方向，如果真有大聪明产出来了顺时针一圈，逆时针一圈的线圈，那么会抵消掉，Flux linkage也不再是简单的$N\phi_m$，具体问题具体分析

Ch13 Faraday's Law and KVL, Inductance

电压表问题

不保守场Non-conservative Field非常的不符合我们的直觉，但是还是能够分析的

Inductance

Self Inductance

Mutual Inductance

Calculating Inductance

Self Inductance

Mutual Inductance

Calculate Inductance using energy principle

[例题] Inductance of Coxial Cable

方法一：用正常方法求

方法二：用能量求

> 直流电的电流会在导线横截面上均匀分布
>
> 而交流电有趋肤效应，频率越高，电流越集中在导线表面
>
> 因此对于交流电，之前积分中间电线的部分要变化
>
> 极高频的交流电，前面那项趋近于0